Habit Persistence, Equilibrium Yield Curve, and the Interest Rate Lower Bound

Kohei Hasui

Aichi University

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Introduction

Model

Numerical Results

Result 1: benchmark case ($\eta = 0, 0.1, 0.15$ and $\alpha = 0$)

Result 2: higher habit persistence under the Taylor rule ($\eta = 0, 0.4, 0.65$)

Result 3: recursive preference under the optimal discretion ($\alpha = -10, 10$)

Conclusion

Introduction

NK models incorporating the zero lower bound (ZLB) constraint on the nominal interest rate:

$$IS(\boldsymbol{\pi}, \boldsymbol{x}, \boldsymbol{i})$$
$$NKPC(\boldsymbol{\pi}, \boldsymbol{x})$$
$$\boldsymbol{i} \geq \boldsymbol{ZLB}$$

- Eggertsson and Woodford, (2003)
- Jung et al.,(2005)
- Coibion et al. (2012)⋯

Stochastic model (incorp. ZLB risk)

- Adam and Billi (2006,2007)
- Nakov (2008)
- Nakata and Schmidt (2019)
- Hills et al. (2019)⋯

➡

Uncertainty effect

- Risky steady state
- Deflationary bias

Monetary policy in a liquidity trap

How does forward guidance affect term premiums in a liquidity trap?

$\Rightarrow$ Nakata and Tanaka (2016) show the effects of forward guidance quantitatively by constructing the NK model formally:

- Incorporating Epstein and Zin (1989)'s (EZ) preference with GHH utility function.
- Reifschneider and Willams (2000)'s rule as a forward-guidance.

State-dependent effect of the ZLB

Nakata and Tanaka (2016) show that term premiums are **state-dependent** when the ZLB is considered:

- Term premiums are constant virtually when the ZLB is not considered.
- On the other hand, term premiums have state-dependence once the ZLB is introduced even though the ZLB constraint is not bind.

Motivation and objective of our paper

Motivation: This paper complements Nakata and Tanaka (2016)'s state-dependence of term premiums from the following three perspectives:

- **Optimal discretionary policy** instead of interest-rate-instrument rule (Taylor rule).
- Introducing **habit formation**, which is incorporated frequently in the literature of asset price.

Objective: This paper examines how the state-dependent effect of the ZLB on the term premium changes.

Habit persistence in NK models

Lagged consumption in IS curve (adhoc) → Modelled in optimization framework $u(C_t, \eta C_{t-1})$

Fuhrer and Moore (1995)

Estimation

Smets and Wouters (2003,2007)

CEE (2005)

Boivin and Giannoni (2006)

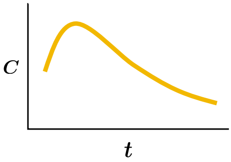
...

Optimal policy

Amato and Laubach (2004)

Leith et al. (2012)

Hump shaped response



$$\min \quad L(\pi, x, \eta)$$
$$\text{s.t.} \quad IS(\pi, x, i, \eta)$$
$$NKPC(\pi, x, \eta)$$

Main findings

- The state-dependent effect of the ZLB on term premium is increased by a small increase in habit persistence under the **optimal discretionary policy**.
- On the other hand, a higher habit persistence decreases the state-dependent effect of the ZLB on the equilibrium yield and term premium decreases under the Taylor rule.

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Model overview

New Keynesian model (RBC + sticky price) which incorporates

- Habit formation (and recursive preference),
- ZLB on the nominal interest rate.

Policy schemes:

- Optimal discretionary policy
- Interest rate rule (Taylor rule with the ZLB)

Model: Households

- Household h's problem (Epstein and Zin, 1989; Rudebusch and Swanson, 2012):

$$\max \quad V_t^h = U(X_t^h, N_t^h) + \beta_t \left[\mathbb{E}_t \left[(V_{t+1}^h)^{1-\alpha} \right] \right]^{\frac{1}{1-\alpha}},$$
$$\text{s.t.} \quad P_t C_t^h + \mathbb{E}_t [M_{t,t+1} B_{t+1}^h] \leq W_t N_t^h + B_t^h + D_t + T_t. \tag{1}$$

$$\text{where} \quad U(X_t^h, N_t^h) = \frac{X_t^{1-\chi_c}}{1-\chi_c} - \varkappa_n \frac{(N_t^h)^{1+\chi_n}}{1+\chi_n}, \quad X_t^h = C_t^h - \eta C_{t-1}$$

$\eta \in [0,1]$ denotes the degree of habit persistence (superficial habits, cf Leith et al., 2012).

- β_t: subjective discount factor:

$$\beta_t = \beta \delta_t, \quad \ln \delta_{t+1} = (1-\rho_\delta) \ln \bar{\delta} + \rho_\delta \ln \delta_t + \sigma_\epsilon^\delta \epsilon_{t+1}^\delta.$$

Model: Households (F.O.C)

First order conditions (symmetric equilibrium):

$$M_{t,t+1} = \beta_t \left[\frac{U_c(X_{t+1}, N_{t+1})}{U_c(X_t, N_t)} \right] \left[\frac{V_{t+1}}{\left[\mathbb{E}_t \left(V_{t+1}^{1-\alpha} \right) \right]^{\frac{1}{1-\alpha}}} \right]^{-\alpha} \frac{1}{\Pi_{t+1}}, \qquad (2)$$

$$\mathbb{E}_t \left[M_{t,t+1} R_t \right] = 1, \qquad (3)$$

$$w_t = -\frac{U_n(X_t, N_t)}{U_c(X_t, N_t)}, \quad U_c(X_t, N_t) = X_t^{-\chi_c}, \quad U_n(X_t, N_t) = N_t^{\chi_n}. \qquad (4)$$

Model: Supply side

- Phillips curve (Rotemberg's sticky price)

$$\left[\varphi\left(\frac{\Pi_t}{\bar{\Pi}}-1\right)\frac{\Pi_t}{\bar{\Pi}}-(1-\theta)-\theta\frac{w_t}{A_t}\right]Y_t = \mathbb{E}_t\left[\varphi M_{t,t+1}\Pi_{t+1}Y_{t+1}\left(\frac{\Pi_{t+1}}{\bar{\Pi}}-1\right)\frac{\Pi_{t+1}}{\bar{\Pi}}\right], \tag{5}$$

- Firm's production function:

$$Y_t = A_t N_t, \tag{6}$$

$$\ln A_{t+1} = (1-\rho_a)\ln \bar{A} + \rho_a \ln A_t + \sigma_\epsilon^a \epsilon_{t+1}^a.$$

- Resource constraint:

$$Y_t = C_t + \frac{\varphi}{2}\left(\frac{\Pi_t}{\bar{\Pi}}-1\right)^2 Y_t. \tag{7}$$

Model: Policy scheme

- Optimal discretionary policy

$$V(\mathbf{s}_t) = \max_{\{\mathbf{z}_t\}} U(X_t, N_t) + \beta_t \Big[\mathbb{E}_t \left[V(\mathbf{s}_{t+1})^{1-\alpha} \right] \Big]^{\frac{1}{1-\alpha}},$$

s.t (8)

Eqs. (2)-(7), and $R_t \geq R_{ZLB}$.

- Taylor rule:

$$R_t = \max \left[R_{ZLB}, \frac{\bar{\Pi}}{\beta} \left(\frac{\Pi_t}{\bar{\Pi}} \right)^{\phi_\pi} \left(\frac{Y_t}{\bar{Y}} \right)^{\phi_y} \right]. \quad (9)$$

Solution method

We solve the full nonlinear stochastic model incorporating the ZLB constraint by Time-Iteration method:

- $\mathbf{z}_t \equiv [C_t, R_t, \Pi_t, Y_t, w_t]^\top$, $\mathbf{s}_t \equiv [\delta_t, A_t, C_{t-1}]^\top \subset \bar{\mathbf{s}}$.
- We compute the policy functions $\mathbf{z}$ as time-invariant functions of $\mathbf{s} \subset \bar{\mathbf{s}}$: The size of state space is given by $N = 41 \times 11 \times 11$.
- Expected terms are approximated with linear spline.
  - The *p.d.f.* of the discount rate shock and productivity shock are assumed to be normal and discretized into 7×7 (jointly) values using the Gaussian quadrature.
  - Maximum Euler residual: 0.025
  - Computing time: about 13 hours (discretion).

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Calibration

Parameters	Description	Values
β	Subjective discount factor	$\frac{1}{1+2.0/400}$
$400 \times (\bar{\Pi} - 1)$	Annualized inflation at the DSS	2
χ_c	Inverse relative risk aversion	0.5
χ_n	Elasticity of firms marginal cost	1
φ	Parameter for price adjustment costs	200
θ	Demand elasticity	10
ϕ_π	Coefficient on inflation in the Taylor rule	3
ϕ_y	Coefficient on output in the Taylor rule	0

Cont...

Calibration (Cont.)

Parameters	Description	Values
ρ_δ, ρ_a	Persistence of exogenous shocks	0.5
σ_e^δ, σ_e^a	Std of disturbance terms	0.5/100
$\bar{\delta}$, $\bar{A}$	Values of exogenous shocks at DSS	1
R_{ZLB}	Interest rate lower bound	1
α	Risk aversion param in recursive pref	$[-10,\ 0^*,\ 10]$
η	Degree of habit persistence	$[0,\ 0.1,\ 0.15^*]$ $[0,\ 0.4,\ 0.65]$

Table: Parameter values. Asterisk $*$ denotes benchmark values.

- η is set considering Chen et al. (2017)'s estimation.
- ρ_δ, ρ_A, σ_e^δ, and σ_e^A are set such that no *extrapolation* are detected.

Numerical results

- We derive policy functions as a function of discount rate (δ_t).
- We fix productivity shock and lagged consumption at the DSS in plotting policy functions:
  - $A_t = \bar{A}$
  - $C_{t-1} = \bar{C}$.

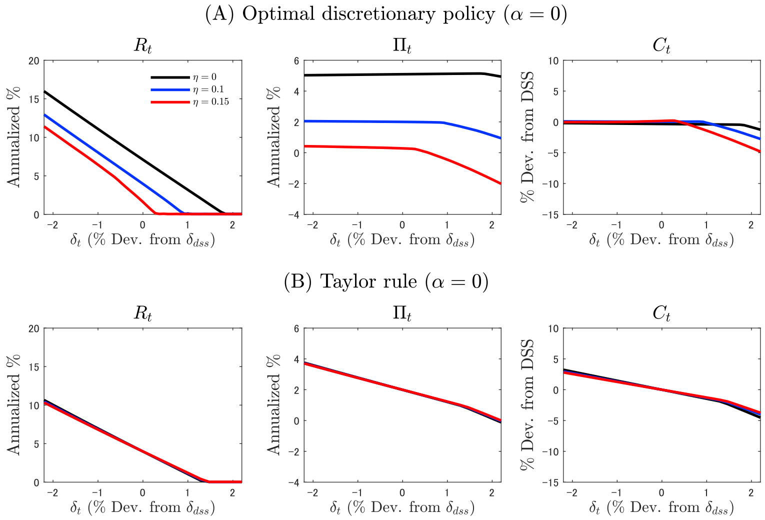

Figure: Policy function of R_t, Π_t, and C_t as a function of discount rate shock δ_t under the optimal discretionary policy (Panel A) and the Taylor rule (Panel B) for degree of superficial habits η. *Note:* The horizontal axis denotes $100 \times (\delta_t - \bar{\delta})/\bar{\delta}$, and we assume that $C_{t-1} = \bar{C}$, $A_t = \bar{A}$, and $\alpha = 0$.

Habit persistence in objective function

- R_t is reduced more aggressively and reaches the ZLB faster as η increases $\Rightarrow$ more severe decreases in Π_t and C_t (Hasui and Hoshino, 2022).
- **Reason:** The static effect of increase in η on utility of consumption is given by

$$\frac{\partial\left(\frac{X_t^{1-\chi_c}}{1-\chi_c}\right)}{\partial\eta_{\mathrm{obj}}} = -X_t^{-\chi_c} C_{t-1} < 0 \quad \text{if} \quad X_t > 0 \text{ and } C_{t-1} > 0, \tag{10}$$

where $X_t = C_t - \eta_{\mathrm{obj}} C_{t-1}$.

The utility of consumption decreases as η_{obj} increases.

$\Rightarrow$ A policymaker has an incentive to reduce nominal interest rates in order to increase consumption utility.

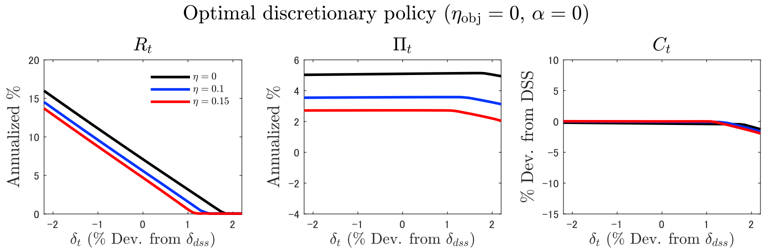

Figure: Policy function of R_t, Π_t, and C_t as a function of discount rate shock δ_t under the optimal discretionary policy when $\eta_{\rm obj} = 0$.

Equilibrium yields and term premiums

- Nominal yield of n-period zero-coupon nominal bond:

$$R_t^{(n)} = -\frac{1}{n} \ln P_t^{(n)}. \tag{11}$$

where $P_t^{(n)}$ denotes bond price:

$$P_t^{(n)} = \mathbb{E}_t \left[M_{t,t+1} P_{t+1}^{(n-1)} \right]. \tag{12}$$

- Nominal term premium of n-period zero-coupon nominal bond:

$$TP_t^{(n)} = R_t^{(n)} - R_t^{(n)Q}. \tag{13}$$

where

$$R_t^{(n)Q} = -\frac{1}{n} \ln P_t^{(n)Q}, \quad P_t^{(n)Q} = \exp\left(-R_t^{(1)}\right) \mathbb{E}_t \left[P_{t+1}^{(n-1)Q} \right]. \tag{14}$$

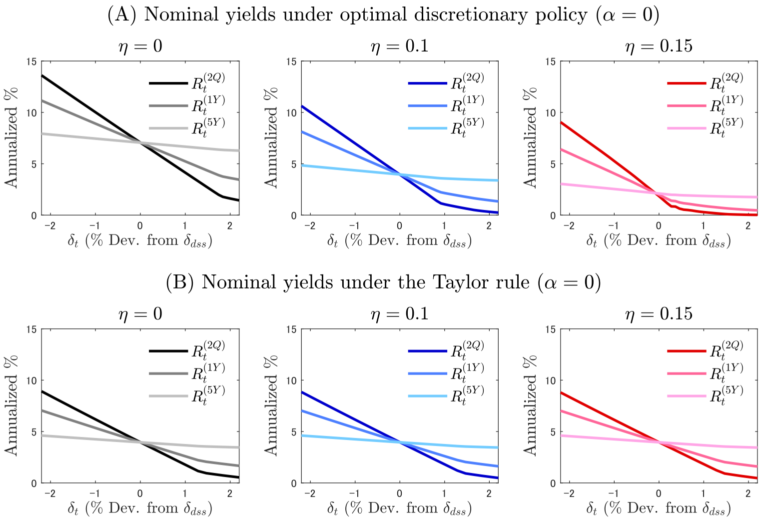

Figure: Policy function of nominal yields under the optimal discretionary policy (Panel A) and the Taylor rule (Panel B) for degree of superficial habits η.

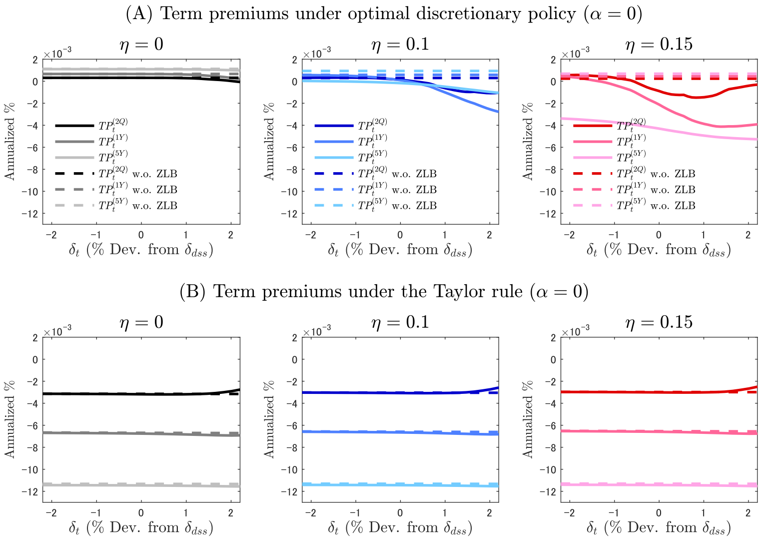

Figure: Policy function of nominal term premiums under the optimal discretionary policy (Panel A) and the Taylor rule (Panel B) for degree of superficial habits η.

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Higher habit persistence

- We assumed quite small degree of superficial habits.
- To study the effect of habit persistence under the Taylor rule, η is set at 0.4 and 0.65.

(A) R_t, Π_t, C_t for $\eta = 0, 0.4, 0.65$ ($\alpha = 0$)

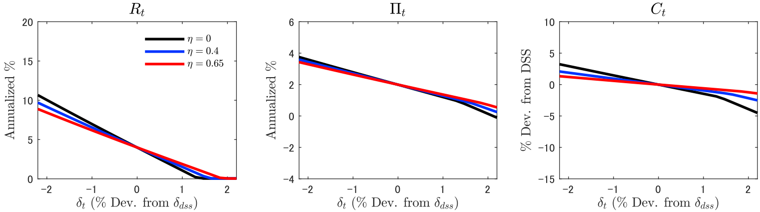

Figure: Policy function of R_t, Π_t, and C_t under the Taylor rule for degree of superficial habits ($\eta = 0, 0.4, 0.65$).

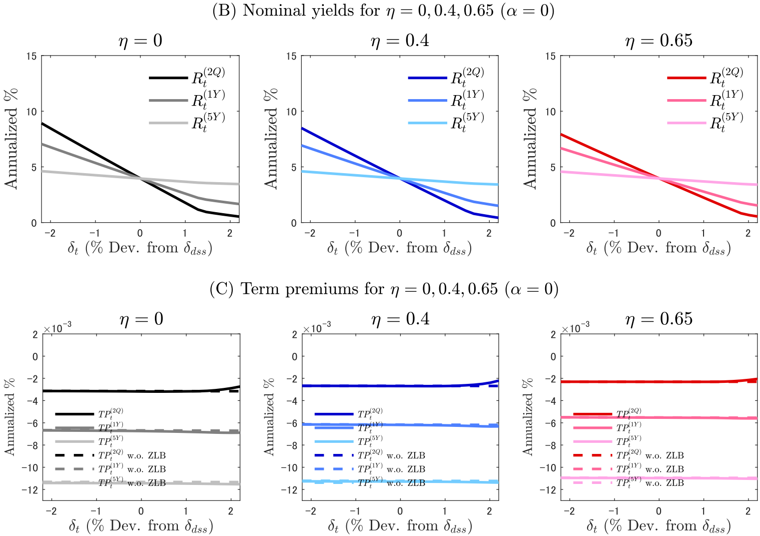

Figure: Policy function of yields (Panels in B), and term premiums (Panels in C) under the Taylor rule for degree of superficial habits ($\eta = 0, 0.4, 0.65$).

Policy implications

- It is pointed frequently that the Taylor rule and optimal discretionary policy are similar in a forward looking and linearized NK models in the literature.
- However, our results show that policy response and policy effectiveness on the macroeconomic variables (especially, term premiums) are quite different by simulating “full non-linear” NK model with the ZLB.
- Chen et al. (2017) showed that estimated values of habit persistence are different with respect to policy scheme

$\Rightarrow$ Our results indicate that the degree of habit persistence is a non-negligible parameter under discretionary policies in terms of the term premium when the ZLB is taken into account.

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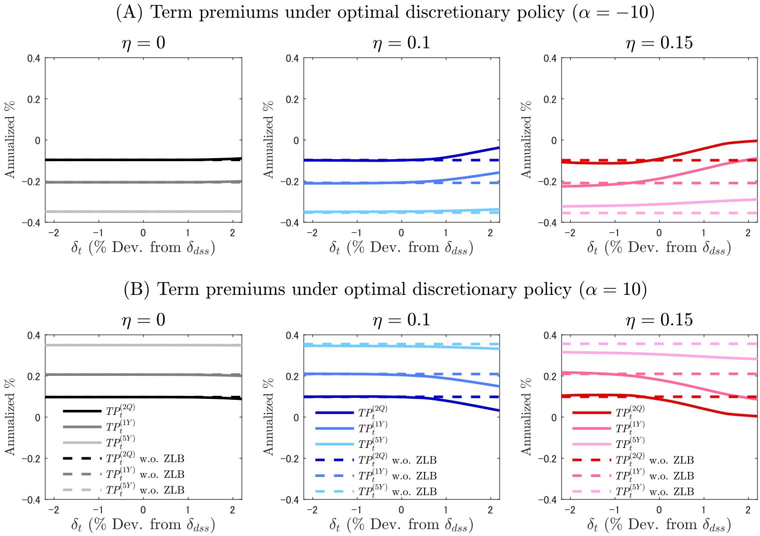

Figure: Policy function of nominal term premiums under the optimal discretionary policy when $\alpha = -10$ (Panel A) and $\alpha = 10$ (Panel B).

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The state-dependent effect of the ZLB on term premium was investigated in the general equilibrium model which incorporates habit formation. The effects of habit persistence were starkly different between the optimal discretionary policy and the Taylor rule:

- **The state-dependent effect of the ZLB on term premium is increased by a small increase in habit persistence under the optimal discretionary policy.**
- On the other hand, a higher habit persistence decreases the state-dependent effect of the ZLB on the equilibrium yield and term premium decreases under the Taylor rule.

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